IC/95/50
hep-ph/9505273

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ON THE FORM FACTORS OF THE $D_s^+ \to \phi\ \mu^+\ \nu_\mu$ DECAY

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Abstract

We apply the infinite mass effective theory, when a heavy quark mass tends to infinity, and Chiral perturbation theory at the quark level, based on the extended Nambu – Jona – Lasinio model with linear realization of chiral $U(3) \times U(3)$ symmetry, to calculate the form factors of the $D_s^+ \to \phi\ \mu^+\ \nu_\mu$ decay up to the first order in current s – quark mass. The theoretical results are compared with experimental data and found to be in good agreement.

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May 1995

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Introduction

In our recent publications [1-3] we considered the form factors of the semileptonic $D \to \bar{K}^*(\bar{K})\, e^+ \nu_e$ decays both in the chiral limit [1,2] and at first order in current s-quark mass expansion [3]. For the description of D-mesons we applied the infinite mass effective theory (IMET) [4,5], when the c-quark mass M_c tends to infinity, i.e. $M_c \to \infty$. In the IMET, we describe the long - distance physics within Chiral perturbation theory at the quark level $(\mathrm{CHPT})_q$ [6], based on the extended Nambu-Jona-Lasinio (ENJL) model with linear realization of chiral $U(3) \times U(3)$ symmetry [7]. The IMET supplemented by $(\mathrm{CHPT})_q$ has been successfully applied to the description of the fine structure of the mass spectra of non-strange $D\,(D^*)$ and strange $D^+_s\,(D^{*\,+}_s)$ and leptonic constants, caused by first order corrections in current-quark mass expansion [8]. The computation of the probabilities of strong and electromagnetic D^* decays performed within IMET and $(\mathrm{CHPT})_q$ has given good results compared with experimental data [9].

In this paper we apply IMET and $(\mathrm{CHPT})_q$ to the calculation of the form factors of the $D^+_s \to \phi\, \mu^+\, \nu_\mu$ decay, keeping corrections up to first order in current s-quark mass.

1. The form factors of the $D^+_s \to \phi\, \mu^+\, \nu_\mu$ decay

The amplitude of the $D^+_s \to \phi\, \mu^+\, \nu_\mu$ decay is determined as follows

$$M\,(D^+_s \to \phi\, \mu^+\, \nu_\mu) \;=\; -\frac{G_F}{\sqrt{2}}\, V^*_{cs}\, <\phi\,(Q)|\bar{s}\,(0)\,\gamma_\alpha\,(1\,-\,\gamma^5)\,c\,(0)|D^+_s\,(p)>\;\ell^{\,\alpha} \quad (1)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi weak coupling constant, $|V_{cs}| = 0.975$ is the CKM mixing matrix element, $s(0)$ and $c(0)$ are the s and c current quark fields with N colour degrees of freedom each, and $\ell^\alpha = \bar{u}(k_{\nu_\mu})\gamma^\alpha(1-\gamma^5)v(k_{\mu^+})$ is the weak leptonic current.

The hadronic matrix element

$$M_\alpha\,(D^+_s\,\to\,\phi) = <\phi\,(Q)|\bar{s}\,(0)\,\gamma_\alpha\,(1\,-\,\gamma^5)\,c\,(0)|D^+_s\,(p)> \qquad (2)$$

can be parametrized in terms of four form factors [3]

$$
\begin{aligned}
M_\alpha\,(D_s^+ \to \phi) \;&=\; i\,a_1\,(q^2)\,\epsilon^*_\alpha\,(Q) \,-\, i\,a_2\,(q^2)\,(\epsilon^*\,(Q)\cdot p)\,(p\,+\,Q)_\alpha \\
&-\; i\,a_3\,(q^2)\,(\epsilon^*\,(Q)\cdot p)\,(p\,-\,Q)_\alpha \\
&-\; 2\,b\,(q^2)\,\varepsilon_{\alpha\beta\mu\nu}\,\epsilon^{*\beta}\,(Q)\,p^\mu\,Q^\nu\,,
\end{aligned} \tag{3}
$$

where q^2 is the square invariant mass of the lepton pair such that $m_\mu^2 \le q^2 \le (M_{D_s^+} - M_\phi)^2$ where $M_{D_s^+}$, M_ϕ and m_μ are the masses of the D_s^+ , ϕ and μ^+ mesons respectively and ϵ^* is the polarisation tensor of the outgoing ϕ meson.

We shall seek the form factors $a_i(q^2)$ $(i\,=\,1,2,3)$ and $b\,(q^2)$ in the form of an expansion in powers of the current s-quark mass upto first order terms

$$
\begin{aligned}
a_i\,(q^2) \;&=\; a_i^{(0)}\,(q^2) \,+\, a_i^{(1)}\,(q^2)\,, \\
b\,(q^2) \;&=\; b^{(0)}\,(q^2) \,+\, b^{(1)}\,(q^2)
\end{aligned} \tag{4}
$$

The form factors $a_i^{(0)}(q^2)$ $(i\,=\,1,2,3)$ and $b^{(0)}\,(q^2)$ are determined in the chiral limit ($ch.\ell.$). They are calculated in the same way as the corresponding form factors for the process $D \to \bar{K}^*\,e^+\,\nu_e$ [1,3], that is

$$
\begin{aligned}
a_1^{(0)}\,(q^2) \;&=\; \sqrt{\frac{3}{8}}\,M_* \\
a_2^{(0)}\,(q^2) \;&=\; \sqrt{\frac{3}{8}}\,\frac{M_*}{M_D^2}\,\Big[\frac{q^2}{M_D^2\,-\,q^2} \\
&+\; \frac{M_D^2\,-\,q^2}{M_*^2}\left(1\,-\,\frac{2\,m\,M_D}{M_D^2\,-\,q^2}\right)\ell n\left(1\,+\,\frac{M_*^2}{M_D^2\,-\,q^2}\right)\Big]\,, \\
a_3^{(0)}\,(q^2) \;&=\; -\sqrt{\frac{3}{8}}\,\frac{M_*}{M_D^2}\,\Big[\frac{2\,M_D^2\,-\,q^2}{M_D^2\,-\,q^2} \\
&-\; \frac{M_D^2\,-\,q^2}{M_*^2}\left(1\,-\,\frac{2\,m\,M_D}{M_D^2\,-\,q^2}\right)\ell n\left(1\,+\,\frac{M_*^2}{M_D^2\,-\,q^2}\right)\Big]\,, \\
b^{(0)}\,(q^2) \;&=\; \sqrt{\frac{3}{8}}\,\frac{1}{M_*}\,\ell n\left(1\,+\,\frac{M_*^2}{M_D^2\,-\,q^2}\right)\,.
\end{aligned} \tag{5}
$$

Here we have denoted $M_* \,=\, \sqrt{M_D\,\bar{v}'/2}$ where $M_D \,=\, 1.86$ GeV is the mass of the charmed pseudoscalar meson at the chiral limit and $\bar{v}' \,=\, 4\,\Lambda \,=$

2.66 GeV. The parameter Λ appears as the cut-off in the Euclidian 3-dimensional momentum space evaluation of constituent quark loop diagrams. This cut-off Λ is connected with the scale of spontaneous breaking of chiral symmetry (SBCS) in $(\mathrm{CHPT})_q$ via the relationship $\Lambda = \Lambda_\chi/\sqrt{2} = 0.67$ GeV at $\Lambda_\chi = 0.94$ GeV [6].

The form factors $a_i^{(1)}(q^2)$ $(i = 1,2,3)$ and $b^{(1)}(q^2)$ are determined by the matrix element [3]

$$\begin{aligned} & M_\alpha^{(1)}(D_s^+ \to \phi) = -i\, m_{0s} \\ & \times \int d^4x < h(Q)|\mathrm{T}(\bar{s}(x)\, s(x)\, \bar{s}(0)\, \gamma_\alpha (1 - \gamma^5)\, c(0))|D_s^+(p) >_{ch.\ell.}. \end{aligned} \tag{6}$$

In accordance with the procedure expounded in [1-3,8,9] we can reduce the matrix element (6) to the expression

$$\begin{aligned} & M_\alpha^{(1)}(D_s^+ \to \phi) = g_D\, m_{0s}\, i\,, \int d^4x \int_{-\infty}^{\infty} dz_0\, \theta(-z_0) \times \\ & \times < \phi(Q)|\mathrm{T}(\bar{s}(x)\, s(x)\, \bar{s}(0)\, \gamma_\alpha \left(\frac{1 + \gamma^0}{2}\right) \gamma^5\, s(z_0, \vec{0}))|0 >_{ch.\ell.} \end{aligned} \tag{7}$$

obtained at leading order in the large N and M_c expansion. The coupling constant g_D has been calculated in [9]

$$g_D = \frac{2\sqrt{2}\,\pi}{\sqrt{N}} \left(\frac{M_D^2}{M_c\, \bar{v}'}\right)^{1/2} \tag{8}$$

The r.h.s. of (7) involves only the light quark fields. Therefore for the evaluation of (7) we can apply $(\mathrm{CHPT})_q$ [1-3,6,8,9]. Since the leading order of the r.h.s. of (7) in current quark mass expansion is fixed by the factor m_{0s}, so the matrix element $< \phi(Q)|\,\mathrm{T}(\ldots)\,|0 >$ has to be calculated in the chiral limit.

In order to evaluate the matrix element (7) let us compare this with the matrix element $M_\alpha^{(1)}(D \to \bar{K}^*)$ describing the $D \to \bar{K}^*$ transition at the first order in current s- quark mass expansion [3]

$$M_\alpha^{(1)}(D \to \bar{K}^*) = g_D\, m_{0s}\, i\,, \int d^4x \int_{-\infty}^{\infty} dz_0\, \theta(-z_0) \times$$

$$\times < \bar{K}^{*}(Q)|\mathrm{T}(\bar{s}(x)\, s(x)\, \bar{s}(0)\, \gamma_{\alpha} \left(\frac{1+\gamma^{0}}{2}\right) \gamma^{5}\, q(z_{0}, \vec{0}))|0>_{\mathrm{ch.l.}} \quad (9)$$

where $q = u$ or d for D^{0} or D^{+}, respectively.

By applying the formulas of quark conversion (Ivanov [6]) one can show that, between matrix elements $M_{\alpha}^{(1)}(D_{s}^{+} \to \phi)$ and $M_{\mu}^{(1)}(D \to \bar{K}^{*})$, there is the relationship

$$M_{\alpha}^{(1)}(D_{s}^{+} \to \phi) = 2\, M_{\alpha}^{(1)}(D \to \bar{K}^{*}) . \tag{10}$$

Readers can verify this relationship by noting that the ϕ meson possesses the quark structure $(\bar{s}\, s)$.

By virtue of the relationship (10) the form factors $a_{i}^{(1)}(q^{2})$ $(i = 1, 2, 3)$ and $b^{(1)}(q^{2})$ read [3]

$$\begin{aligned}
a_{1}^{(1)}(q^{2}) &= \sqrt{3}\, \frac{m_{0s}}{M_{*}}\, \frac{\bar{v}}{4m}\, M_{D}\, \ell n \left(\frac{\bar{v}'}{4m}\right) \\
a_{2}^{(1)}(q^{2}) &= -\, a_{3}^{(1)}(q^{2}) = b^{(1)}(q^{2}) \\
b^{(1)}(q^{2}) &= \sqrt{3}\, \frac{m_{0s}}{M_{*}}\, \frac{\bar{v}}{4m}\, \frac{M_{D}}{M_{D}^{2} - q^{2}} \left[1 - \ell n \left(1 + \frac{M_{*}^{2}}{M_{D}^{2} - q^{2}}\right)\right] .
\end{aligned} \tag{11}$$

Now we can get the numerical values of the form factors at $q^{2} = 0$

$$\begin{aligned}
a_{1}(0) &= a_{1}^{(0)}(0) + a_{1}^{(1)}(0) = 0.96 + 0.28 = 1.24\, (\mathrm{GeV}) , \\
a_{2}(0) &= a_{2}^{(0)}(0) + a_{2}^{(1)}(0) = 0.14 + 0.06 = 0.20\, (\mathrm{GeV})^{-1} , \\
a_{3}(0) &= a_{3}^{(0)}(0) + a_{3}^{(1)}(0) = -\, 0.42 - 0.06 = -\, 0.48\, (\mathrm{GeV})^{-1} , \\
b(0) &= b^{(0)}(0) + b^{(1)}(0) = 0.21 + 0.06 = 0.27\, (\mathrm{GeV})^{-1} .
\end{aligned} \tag{12}$$

One finds that the first order corrections in current s- quark mass expansion are between 14 and 43%. The form factors $a_{i}(q^{2})$ $(i = 1, 2, 3)$ and $b(q^{2})$ are connected with the standard form factors $A_{i}(q^{2}) =$ $(i = 1, 2, 3)$ and $V(q^{2})$ via the relations [1]

$$
\begin{aligned}
A_1(0) &= \frac{1}{M_{D_s} + M_\phi} a_1(0) = 0.43 \\
A_2(0) &= (M_{D_s} + M_\phi)\, a_2(0) = 0.60 \\
A_3(0) &= (M_{D_s} + M_\phi)\, a_3(0) = -1.44 \\
V(0) &= (M_{D_s} + M_\phi)\, b(0) = 0.81
\end{aligned}
\tag{13}
$$

where $M_{D_s} = 1.97$ GeV and $M_\phi = 1.02$ GeV [10].

The theoretical values are in good agreement with the experimental data [11,12]

$$
\begin{aligned}
(R_v)_{\rm th} &= \frac{V(0)_{\rm th}}{A_1(0)_{\rm th}} = 1.9, \qquad (R_v)_{\rm exp} = \left\{ \begin{array}{l} 1.8 \pm 0.9 \pm 0.1\ [11] \\ 1.4 \pm 0.5 \pm 0.3\ [12] \end{array} \right. \\
(R_2)_{\rm th} &= \frac{A_2(0)_{\rm th}}{A_1(0)_{\rm th}} = 1.4, \qquad (R_2)_{\rm exp} = \left\{ \begin{array}{l} 1.1 \pm 0.6 \pm 0.1\ [11] \\ 0.9 \pm 0.6 \pm 0.3\ [12] \end{array} \right.
\end{aligned}
\tag{14}
$$

Conclusion

By applying IMET supplemented by $(\mathrm{CHPT})_q$ we have evaluated the form factors of the $D_s^+ \to \phi\,\mu^+\,\nu_\mu$ decay upto first order in current s-quark mass. The theoretical predictions compare reasonably well with experimental data. The proposed approach allows us to set up a relationship (10) between chiral corrections to the form factors of the decays $D_s^+ \to \phi\,\mu^+\,\nu_\mu$ and $D \to \bar{K}^*\,e^+\,\nu_e$. Unfortunately, the possibility of the experimental investigation of the relationship (10) goes beyond the available accuracy of present day experimental abilities.

Acknowledgements

ANI and NIT would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. We acknowledge with pleasure fruitful discussions with Prof. G. E. Rutkovsky.

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